MathAmigos and SFPS Grades 4-6 Workshop
Math Festival Activities: A Round Robin

Session Leaders: James Taylor, *Math Circles Collaborative of New Mexico & MathAmigos*
Shannon Rogers, *Art of Problem Solving & MathAmigos*
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Agenda & Packet Contents

1. Intro to Math Circles
2. Math Circle Pledge
3. The Math Salute
4. What is a Julia Robinson Math Festival?
    a. Video at http://jrmf.org/
    b. NM festivals: SFCC, AIMS charter, San Juan College (upcoming Pi Day), San Felipé Pueblo Day School
    c. Family math nights are similar. Will you help us as a table leader February 6 or 21? **You will already be an "expert" on some activities!**
5. The Festival!
    a. What are your goals for this session?P
    b. Groups of 2-4. Consider staying with the same group or reforming groups. Advantages? Disadvantages?
    c. 2-3 different activities during this session
    d. What is a table leader and the role of one?
    e. For afterwards: "Table leader guide", abridged
    f. Play! Discuss!
    g. 20 minutes or so per activity.
6. Discussion, Q & A
    a. ***Especially*** how would you implement during the school day or after school?
7. Festival Math and Common Core: There are too many activities here to list this out in detail. As (and after)you investigate your chosen activities, please reflect on what Common Core alignments might be applicable.
8. Handout: List of activities
9. Handout: A table leader's (abridged) guide. The role of a table leader.
10. Activity Handouts: Please take a copy of any activity sheets from a bin—**except for laminates**. We can also provide you with pdfs on the mathamigos.org website.

Some Resource Links

- Julia Robinson Mathematics Festivals (source of festival problems): http://jrmf.org
- Math Pickle: http://mathpickle.com/ (math activities and games by grade level)
- AMS/MSRI Mathematical Circles Library: http://bookstore.ams.org/MCL
- Math Teachers' Circles Network: https://www.mathteacherscircle.org/
- MTCircular magazine: http://www.mathteacherscircle.org/news/mtc-newsletter/
- National Association of Math Circles: They also have run and will be running math circle workshops at the University of Colorado at Denver with grant money attached.: https://www.mathcircles.org/

- From Russia with Math (for Kids), Scientific American article[1]
- My email: jtaylor@mathcirclesnm.org

Great web resources from James Tanton (James has lots of very short videos suitable for showing to students being introduced to problem solving):

- http://JamesTanton.com
- http://gdaymath.com
- http://www.maa.org/math-competitions/teachers/curriculum-inspirations
- Consider doing Global Math Week in October! https://www.theglobalmathproject.org/gmw

Festival activities & materials, MathAmigos Workshop, 26 Jan 2019

25-Jan-2019

Workshop Activities

#	Activity	1st Filenames/PDFs/links	2nd Filenames/PDFs/links	Physical items/manipulatives
1	Color Triangle Challenge	Color Triangle.pdf		3 colors of poker chips
2	ConHex	ConHex 41-cell board, full page, not numbered.pdf	ConHex instructions.pdf	Tub: boards and colored markers
3	Cookie Monster	Cookie Monster.pdf	Cookie Monster worksheets.pdf	Tub: pennies
	Criss-Cross	Criss Cross.pdf		
4	Difference Engine	Difference Engine-Full Instructions.pdf	DifferenceEngineDiagram.pdf	
5	Estimation Games	jrmf estimation games.pdf		Tub: pennies
24	More Nim Games			
19	Palindrome Grab	PalindromeGrab.pdf		2 colors of poker chips
	Polyhedra Challenge	Polyhedra Challenge-2018 NNMC Summer Math Workshop		2-3 tubs of Polydrons, dry-erase boards
	Squaring and subtracting puzzles	Squaring-and-subtracting-with-intro.pdf		
23	The 15 Game	The Game of 15-NEW.pdf		Tub: Post-its, numbered
18	Towers of Hanoi	Towers of Hanoi.pdf		Tub: Three tower boards and washers

[1] https://blogs.scientificamerican.com:budding-scientist:from-russia-with-math-for-kids:%3FWT.mc_id=SA_syn_BusinessInsider

A Math Festival Table Leader's Guide (abridged)

Overview
You will be "leading" a table—one of two adjacent tables running the same math activity. There will be from two to four of you at each of those pairs of tables. Students will circulate throughout the room until they find an activity that interests them. Feel free to tout your activity!

The activities are designed to be easy to start (low threshold), but most of them get very deep (high ceiling)—hard enough that the activity designers don't know the answers to the final questions.

Your own investigation
We will give your activity team a tub containing your activity, instructions, and materials/manipulatives. Dive in! If something is unclear to you, then ask away! We may or may not help you at first. We do need to know if something about the instructions is unclear or if there are *missing* materials/manipulatives. And…**no devices** (for you or the students at your table)! There is nothing here that requires them, and they are counterproductive.

Can you imagine doing this—your table activity—with a constantly changing group of students for two hours?

Did anyone dominate your group? How would you manage this with the students are your table?

How to be a JRMF table leader
<u>The dance of the JRMF table leader</u>
If you are a teacher at any level you may find the role of table leader at a JRMF table (or as a math circle leader someday!) a bit of a change of pace:

Your role is not to teach anything at all!

Whoa. Even though you will have explored the activity in your training session, you will likely feel unprepared to teach it. So no worries. *That is your ideal condition*.

A big part of your job is to help students both stay engaged and to *have a good time*. Our overall goal is to keep students "productively stuck" on the activity, rather than to get them through the activity and to whatever answers they can come up with as quickly as possible. If they are engaged, you have very little to do—a good thing. If they are stuck or frustrated, your job is to get them moving again without giving away too much. And to help them see the puzzle as a chance to play. Remind them that problem solving need not be a solitary game—enlist your tablemates. Suggestions for how to do all this are in the next section.

Your motto: **Be less helpful**

During the festival, should your table be well-covered by your team, feel free to take a break and visit other activity tables. If it seems appropriate, consider asking the students to explain what their activity is and what they have discovered.

Asking students good mathematical questions

The Julia Robinson Math Festival session leader Guide provides some great points about this...

- *The kind of communication you do depends a lot on the kid. For some, you'll be helping them read and understand the questions.*
- *For a more advanced student, you'll be reminding them of key strategies like*
    - *Can you try a simpler example?*
    - *Can you do a related problem?*
    - *Draw a picture*
- *Maybe they already solved the problem, and you want to guide them toward a different way of looking at it. Generally, try to be the one asking questions more than answering them.*
    - *Can you explain that to me?*
    - *Why does that have to be so?*
    - *Can you show me an example?*
- *It is also good to be prepared with some questions to ask when kids make a mistake or get something completely wrong.*
    - *Try to say "Why?" with the same tone whether they have it right or wrong -- make them explain it to you and uncover their own error.*
    - *Talk about ways they can check their answer, again whether it is right or wrong.*
- *If you hear another kid putting someone down for not knowing an answer or for getting something wrong, be prepared with a remark that suits you. I like to remind them that we're here to explore and learn, and it would be boring if we already knew how to do all the problems.*

Teach students how to ask good mathematical questions

From Sam Vandervelde's *Circle in a Box:*

Students almost never have the opportunity to ask, "What if . . . ?" types of questions at any point during their secondary school careers. They have no idea that one of the most crucial skills acquired by a professional mathematician is the ability to ask productive questions; the sorts of questions that lead to new areas of research. All of their training suggests that mathematics is synonymous with solving problems; very few of them stop to wonder where the problems come from. Moreover, working on problems can become tiresome or frustrating. But contemplating new directions to explore, free from the burden of needing to answer all the questions that might arise (at least for the time being), is a marvelous, creative endeavor. Every student should be given the chance to practice this process.

So take a few minutes after wrapping up a nice problem to point out that the book is not yet closed on this particular idea, and ask students where it might lead. At first students may need a lot of coaxing. What happens if one uses different numbers or shapes? Is there an analogous result in higher dimensions? What if we allowed three people to play this game, how would that look? Encourage any ideas or attempts; quite often once the first question or two is tentatively offered the floodgates are opened. Help students refine vague ideas into well formulated questions. This activity can be as rewarding for the leader as for the students— it

is exciting to see what they come up with, and invariably everyone leaves with new ideas to pursue.

<u>And again…</u>
Don't try to teach! Enjoy seeing how quickly young minds play with new activities and go off in all sorts of crazy directions. As adults, we often try to channel these wild speculations, but who knows where they may lead an inquisitive young mathematician. I have often found middle schoolers to be faster at gaining insights or in finding surprising lateral directions to explore than older kids or adults.

Math Circle Problem-Solving Strategies

Compiled by Bay Area Math Circle session leader Emily McCullough, with assistance from Josh Zucker and Tom Davis.
http://bact.mathcircles.org/files/Summer2013/Problem%20Solving%20Strategies.pdf

- **Do something. Get your hands dirty. Play.**
- **Patience.**
  Remember the difference between a problem and an exercise. *This is a problem.*
- **Start small and simple.**
  Build from there. it is almost always useful to look at the very smallest versions of a problem. They are a lot easier to work out, usually.
- **Special and Specific Cases.**
  - Look at specific cases or particular configurations.
  - Work out a specific example or play the game a few times. See what happens...
  - Look for extremes.
  - Make and solve an easier problem.
- **Organization.**
  - If you are working on a problem that can be split into cases, make sure you have got all the cases.
  - If you are looking at examples, or doing experimental calculations, keep track of this information in an organized way.
  - Experiment with multiple ways of recording and presenting your findings. One picture or graph or table may be better than another for seeing patterns or visualizing your results.
- **Look for a pattern.**
  Start small and simple here too. Patterns and formulas that hold for these specific cases often provide a clue for the general case.
- **Generalize.**
  Sometimes generalizing makes a problem easier. Variables are not always scarier or harder to work with. And you can prove something more with less!
- **Symmetry.**
- **Wishful Thinking.**
  Factor $x^4 + x^2+1$. This seems difficult, but it would be easy if you had $2x^2$ instead of x^2. So just change it to what you want (but correct it too):
  $$x^4+x^2+1= x^4+2x^2-x^2+1= (x^2+1)^2 -x^2$$
  But that is just the difference of two squares, so:
  $$x^4 + x^2 + 1= (x^2+1-x)^2 (x^2+1+x)^2$$
- **Work Backwards.**
- **Recycle. (Reduce, Reuse, Recycle)**
  If you have worked out values for simpler versions of the problem, perhaps you can use them to work out harder versions.
- **Give things names.**
  Give a name or symbol to new objects and operations you are using. Also name the mathematical properties and objects you use and observe while problem-solving (e.g. commutative property of addition, divisors, parity). This is a great opportunity to use student names to name theorems/ideas by the Math Circle group, ex: Sam's Theorem.

13. **Make a picture.**
14. **Invariants. Monovariants.**
    Sometimes you can find a quantity that is unchanged after every operation (called an invariant) and sometimes you can find a quantity that changes in one direction (called a monovariant).