Julia Robinson Mathematics Festival

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Switching Light Bulbs

A long hallway has 1000 light bulbs with pull strings, numbered 1 through 1000. If the light bulb is on, then pulling the string will turn it off. If the light bulb is off, then pulling the string will turn it on. Initially, all the bulbs are off.

At one end of the hallway, 1000 people numbered 1 through 1000 wait. Each person, when they walk down the hallway, will pull the string of every light bulb whose number is a multiple of theirs. So, for example, person 1 will pull every string; person 2 will pull the strings of bulb number 2, 4, 6, 8, 10, …, and person 17 will pull the strings of bulb number 17, 34, 51, 68, … .

For each situation below, which light bulbs are on after all the indicated people are done walking?

1. Everyone
2. The evens, or in other words, all the people whose numbers are even.
3. The odds
4. The primes
5. The perfect squares
6. The multiples of 3
7. The perfect cubes
8. The people 1 more than a multiple of 4.
9. The people 2 more than a multiple of 4 (that is, the evens not divisible by 4).
10. Any other interesting sets you'd like to consider?
11. Given the set of people who walked, what is a general strategy for figuring out which light bulbs are turned on?

For each situation below, which people should walk in order for the indicated sets of light bulbs to end up being the only ones turned on?

12. All the bulbs.

13. The odds, or in other words, all the light bulbs whose numbers are odd.

14. The evens

15. The primes

16. The perfect squares

17. The perfect cubes

18. The multiples of 3

19. The multiples of 4

20. The multiples of 6

21. Any other interesting sets you'd like to consider?

22. Given the set of light bulbs that are turned on, what is a general strategy for figuring out which people walked?

23. For any set of light bulbs, does there necessarily exist a set of people who can walk such that the given set of light bulbs ends up being the only set turned on? If so, prove it. If not, describe the sets of light bulbs that are impossible.

24. Suppose that there are still 1000 people, but there are more than 1000 light bulbs. Not knowing which people walked, but only knowing which of the first 1000 light bulbs are turned on, what can you predict about which of the bulbs beyond #1000 are turned on?

Thanks to Stan Wagon's Macalester problem of the week for the idea behind this extension of the famous locker problem. Thanks to Glenn Trewitt and Car Talk for the idea of using light bulbs instead of lockers.