

Primes in Evenland

In Evenland, the citizens never managed to invent the number one; they only invented 2. So then they figured out that they could add, and using $2 + 2$, they made 4. Later they kept using addition and created the numbers 6, 8, 10, and so on. Eventually they started multiplying, and found for example that 4×6 is 24, so 24 is not prime; an Evenland prime number is one that can't be made by multiplying two numbers in Evenland.

1. Is 2 prime? Why or why not?
2. Is 4 prime? How about 6, 8, 10, and 12 — are they prime?
3. Keep going for a while, classifying numbers as prime or composite. Is there a pattern you can recognize, so you could easily determine whether 1234874 is prime in Evenland? Can you explain why the pattern occurs?

In our normal number land, if you start with some number like 48 and keep factoring (for instance, making a factor tree), it doesn't matter whether you start out breaking the 48 into 4×12 or into 6×8 . In the end, you'll find $48 = 2 \times 2 \times 2 \times 3$, maybe with those numbers written in a different order, but in any case with the same list of numbers. This is called “Unique Factorization” or sometimes “The Fundamental Theorem of Arithmetic”. But in Evenland, this isn't true!

4. Find a number that has two different prime factorizations. What is the smallest such number?
5. Describe a pattern you can use to quickly identify Evenland numbers that have two different prime factorizations. Why does this pattern work?
6. Can an Evenland number have more than two prime factorizations? Could you find a number that has 5 different factorizations? 8 different factorizations? Is any number of factorizations possible?
7. Describe the relationship between our normal-land prime factorization and the number of prime factorizations a number has in Evenland.
8. With ordinary integers, there's a convenient property that is very closely related to the Fundamental Theorem of Arithmetic: if a product is divisible by a prime, then one of the factors must be divisible too. Does this work in Evenland?
9. For a much bigger challenge, try traveling to some other lands, like square root of 2 land, where the numbers are things like $3 + 7\sqrt{2}$ — now what are the primes? Is there unique factorization? What if we use different square roots in place of 2?